
Universidad Adolfo Ibáñez
Facultad de Ingeniería y Ciencias

# Dynamics of Automata under different update schedules

Ph.D. Thesis

Author: Stephanie MacLean Konga

Advisor: Eric Goles Chacc

Coadvisor: Marco Montalva Medel

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*Y poderoso es Dios para hacer que abunde en vosotros toda gracia, a fin de que, teniendo siempre en todas las cosas todo lo suficiente, abundéis para toda buena obra.*
*2 Corintios 9:8*

# Abstract

In this thesis our main interest is focused specifically on the dynamics of cellular automata. Dynamics will be addressed by the *invariance* of the automaton, and in some cases invariance will be addressed by its *reversibility*. Our concept of invariance will be considered under the set of attractors of the automaton, i.e. it's periodic configurations. This concept suggests to question the *robustness* of the automaton, this is, the stability of the behaviour regardless external disturbances, such as different update schedules. This is important in dynamical systems, in order to determine and prove strong properties that are invariant under structural modifications.

More precisely, we have studied the *block invariance* and *attractor invariance* of the elementary cellular automata, Section 3.1 and 3.2 respectively, and *invariance* of linear rules with radius 2, Section 4.2. On the one hand, we have studied 11 conjectures about block invariance, that were previously established in [7]. We were able to prove 9 of them and refute the other 2 left. Also, for all 256 elementary cellular automata we established equivalences in between them by means of the configurations of their set of attractors. In the case of attractor invariance we managed to characterize the set of attractor of the elementary cellular automata rules under sequential update schedules, so to establish equivalences (classes) in between these rules by means of the configurations of their set of attractors. We have proven 2 of these classes, leaving the rest of them as future work. On the other hand, we were able to characterize the update schedules for which the linear rules 90 and 150 are invariant, and the same was done for linear rules with radius 2. The key tool to prove invariance for linear rules was the study of the reversibility of each rule.

Due to this work we have published two articles, [8, 11].

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